Exercise 43

A particle moves along a straight line with equation of motion $s = f(t)$, where s is measured in meters and t in seconds. Find the velocity and the speed when $t = 4$.

$$f(t) = 80t - 6t^2$$

Solution

The velocity is the derivative of $s = f(t)$.

$$\begin{aligned}
f'(t) &= \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \\
&= \lim_{h \to 0} \frac{[80(t+h) - 6(t+h)^2] - [80t - 6t^2]}{h} \\
&= \lim_{h \to 0} \frac{[80(t+h) - 6(t^2 + 2th + h^2)] - 80t + 6t^2}{h} \\
&= \lim_{h \to 0} \frac{(80t + 80h - 6t^2 - 12th - 6h^2) - 80t + 6t^2}{h} \\
&= \lim_{h \to 0} \frac{80h - 12th - 6h^2}{h} \\
&= \lim_{h \to 0} (80 - 12t - 6h) \\
&= 80 - 12t
\end{aligned}$$

Therefore, the velocity when $t = 4$ is

$$f'(4) = 80 - 12(4) = 32 \ \frac{\text{m}}{\text{s}},$$

and the speed when $t = 4$ is

$$|f'(4)| = |32| = 32 \ \frac{\text{m}}{\text{s}}.$$